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ABSTRACT

One of the greatest challenges for mathematics teacher educators committed to reforming and improving mathematics education is to help preservice elementary and middle school teachers develop an appreciation for algebraic reasoning. Preservice teachers' views of algebra are typically derived from their experiences in middle school and high school where they developed a conception of algebra as a body of rules and procedures for manipulating symbols. However, as Moses (1997) observed, the content of algebra is being transformed from a discipline involving the manipulation of symbols to a way of seeing and expressing relationships, "a way of generalizing the kinds of patterns that are part of everyday activities" (p. 246). In elementary and middle school, the crucial issue now appears to be the development of algebraic reasoning, with a focus on relationships, not just the introduction of algebraic concepts (Yackel, 1997). An important goal for mathematics teacher educators is to organize experiences for preservice teachers that will broaden their vision of algebra so that they can effectively promote the algebraic reasoning of elementary and middle school children. This investigation focused on an algebra course for preservice elementary and middle school teachers who have chosen mathematics as their area of specialization. The goals of the course are (1) to develop preservice teachers' understanding of algebraic concepts, (2) to encourage preservice teachers to focus on children's algebraic reasoning, and (3) to foster more conceptual views about the nature of algebra. The research mission of this investigation was to assess progress toward meeting the goals of the course. (Author)

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PRESERVICE ELEMENTARY AND MIDDLE SCHOOL TEACHERS' CONCEPTIONS OF ALGEBRA REVEALED THROUGH THE USE OF EXEMPLARY CURRICULUM MATERIALS

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One of the greatest challenges for mathematics teacher educators committed to reforming and improving mathematics education is to help preservice elementary and middle school teachers develop an appreciation for algebraic reasoning. Preservice teachers' views of algebra are typically derived from their experiences in middle school and high school where they developed a conception of algebra as a body of rules and procedures for manipulating symbols. However, as Moses (1997) observed, the content of algebra is being transformed from a discipline involving the manipulation of symbols to a way of seeing and expressing relationships, "a way of generalizing the kinds of patterns that are part of everyday activities" (p. 246). In elementary and middle school, the crucial issue now appears to be the development of algebraic reasoning, with a focus on relationships, not just the introduction of algebraic concepts (Yackel, 1997). An important goal for mathematics teacher educators is to organize experiences for preservice teachers that will broaden their vision of algebra so that they can effectively promote the algebraic reasoning of elementary and middle school children.

This investigation focused on an algebra course for preservice elementary and middle school teachers who have chosen mathematics as their area of specialization. The goals of the course are (1) to develop preservice teachers' understanding of algebraic concepts, (2) to encourage preservice teachers to focus on children's algebraic reasoning, and (3) to foster more conceptual views about the nature of algebra. The research mission of this investigation was to assess progress toward meeting the goals of the course.

Recommendations from various professional organizations (CBMS, 2001; MAA, 1991; NCTM, 1991) have outlined the algebraic content appropriate for preservice teachers. This content includes investigating patterns, representing problem situations with variables, analyzing functional relationships, and investigating algebraic structure. Although it may be difficult to address all of these topics meaningfully in a one-semester course, we decided to use this collection of topics as the framework for the content of our course (Stump & Bishop, 2001). As we address these topics, we wish to shift the emphasis away from symbolic manipulations and toward a broader and more relationship-oriented view of algebra.

Bednarz, Kieran, and Lee (1996) suggested four conceptual approaches for developing algebraic ideas with children – generalization, problem solving, modeling, and

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functions – and these four approaches also offer promising alternatives for developing algebraic ideas with preservice teachers. As described by Bednarz, Kieran, and Lee, *generalization* focuses on the construction of formulas that account for general procedures or relationships among quantities. *Problem-solving* emphasizes the forming and solving of equations, using letters as unknowns. *Modeling* involves algebraic representations arising out of real-world situations, and relationships originating in observations or measurements. A *function* approach examines various representations of dependence relationships among real-world quantities, observing how change in one variable produces variation in another variable. Each of these approaches provides a different setting in which the focus is on the development of conceptual knowledge and algebraic reasoning. For our purposes, we have collapsed these four approaches into three: generalization, problem solving, and a combination of functions and modeling. In our view, modeling a physical situation frequently involves identifying the function that explains the situation, and the emphasis on relationships that is inherent in work with functions may help interpret a situation that is being modeled. The three approaches provide a means of organizing algebra content that goes beyond building up layers and layers of procedures.

Although several conceptually-oriented textbooks exist for developing algebraic ideas with children, analogous materials for use with preservice teachers are not easy to find. Algebra textbooks do exist for college students, but they are typically designed to prepare students for subsequent mathematics courses, not to prepare preservice teachers for the mathematics classroom, and they do not necessarily contain the topics in our framework. Lacking a tailor-made textbook, we resolved the issue by selecting a nontraditional college algebra textbook (one that emphasizes a functions-and-modeling approach to algebra) and then supplementing it with exemplary elementary and middle school curriculum materials, including algebra modules from *Connected Mathematics* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998a, b, c, and d) and *Mathematics in Context* (National Center for Research in Mathematical Sciences Education & Freudenthal Institute, 1998).

This investigation focused on the role of these exemplary curriculum materials in serving the goals of our algebra course for preservice teachers. The research addressed the following questions: (1) What aspects of preservice teachers' knowledge of algebraic concepts are revealed in their use of exemplary curriculum materials designed for middle school students? (2) What aspects of preservice teachers' focus on children's algebraic reasoning are revealed in their use of these materials? and (3) What are preservice teachers' views about the nature of algebra before and after the course?

Method

The participants in this investigation were 30 elementary education majors enrolled in an algebra course for preservice elementary and middle school teachers who had chosen mathematics as their area of specialization. Five of these preservice

teachers also indicated plans to complete a middle school mathematics endorsement. All of the preservice teachers had successfully completed two years of high school algebra and 13 had successfully completed a year of high school calculus. All of the preservice teachers were juniors or seniors and had completed the two-course sequence of mathematics required of all elementary education majors. Only two of the preservice teachers had taken a mathematics teaching methods course. For the 26 preservice teachers who had taken the SAT exam, the median mathematics score was 560.

On the first day of class, the preservice teachers wrote their responses to the question, "What is algebra?" They answered the same question again on the last day of class. The class spent the first four weeks investigating patterns and solving problems selected from the NCTM Algebra *Navigations* series (Cuevas, Yeatts, & House, 2001), Driscoll's (1999) *Fostering Algebraic Thinking: A guide for teachers, grades 6-10*, and algebra modules from *Connected Mathematics* (Lappan et al., 1998c) and *Mathematics in Context* (National Center for Research in Mathematical Sciences Education & Freudenthal Institute, 1998).

In the second week of the course, the preservice teachers read and discussed the article, "A Foundation for Algebraic Reasoning in the Early Grades" (Yackel, 1997). This article describes a classroom discussion in which at least one student moves from a focus on isolated cases to reasoning about the entire range of possibilities. Then each preservice teacher wrote responses to a series of questions.

During this time, the class spent three days exploring *Comparing Quantities*, a sixth-grade algebra module from *Mathematics in Context* (National Center for Research in Mathematical Sciences Education & Freudenthal Institute, 1998). After working through sections of the module in groups, the preservice teachers completed the Chickens problem (p. 21) individually as an in-class quiz. The following day, the class examined the various strategies used to solve the Chickens problem. Then they received an interview assignment to pose the Chickens problem to a child in grades 4-8 and to ask questions to elicit as much information as possible about the child's mathematical thinking. Each preservice teacher then completed a written analysis of his or her interview with the child.

The class then spent three days exploring various sections of *Say It with Symbols: Algebraic Reasoning*, an algebra module for grade 6 from the *Connected Mathematics* series (Lappan et al., 1998c). Preservice teachers completed Problem 3.3: Finding the Area of a Trapezoid and follow-up questions (pp. 40-41) as a quiz.

In the fifth week, the class began a ten-week exploration of functions and modeling. Throughout this time, the class alternated its focus between selected investigations of linear, exponential, and quadratic functions in *Connected Mathematics* algebra modules and corresponding lessons in the college algebra textbook. They investigated linear relationships in *Thinking with Mathematical Models* (Lappan et al., 1998d), exponential relationships in *Growing, Growing, Growing* (Lappan et al., 1998b), and

quadratic relationships in *Frogs, Fleas, and Painted Cubes* (Lappan et al., 1998a). The class spent two or three days with each module and completed a quiz after each one.

The last three weeks of the semester were devoted to matrices and investigations of algebraic structure. This investigation does not focus on that part of the course.

Research Findings

Preservice Teachers' Understanding of Algebraic Concepts

For this investigation, samples of the preservice teachers' work were collected to reflect each of the three conceptual approaches to algebra adapted from Bednarz, Kieran, and Lee (1996): generalization, problem solving, and functions and modeling. The data sources for examining preservice teachers' understanding of algebraic concepts included: (1) solutions to the Area of a Trapezoid problem, (2) solutions to the Chickens problem, and (3) solutions to quizzes focusing on linear, exponential, and quadratic functions.

Generalization

Generalization is described by Bednarz, Kieran, and Lee (1996) as focusing on the construction of formulas that account for general procedures or relationships among quantities. The Area of a Trapezoid problem in *Say it With Symbols* (Lappan et al., 1998c) provides an opportunity to assess preservice teachers' reasoning about relationships among geometric quantities. This problem asks students to interpret four hypothetical children's methods for finding the area of a trapezoid. Table 1 shows preservice teachers' success with the various tasks associated with the problem. Task 4 presented the greatest challenge. Preservice teachers struggled with creating a drawing to match the expression $\frac{1}{2}h(b-a)+ha$ to illustrate finding the area of a trapezoid.

Problem Solving

Bednarz, Kieran, and Lee (1996) define the problem-solving approach as solving problems by forming and solving equations using letters as unknowns. The Chickens problem, from the sixth-grade *Comparing Quantities* (National Center for Research in Mathematical Sciences Education & Freudenthal Institute, 1998), depicts a situation involving three unknown quantities, the weights of three different chickens. This problem asked students to examine a series of three drawings, each showing a different combination of two of the three chickens in the problem. Given the total weight for each pair of chickens, the problem is to find the weight of each individual chicken. A possible solution strategy is to set up and solve a system of three equations involving three variables. Sixth-graders, though, would be more likely to guess and check or to employ informal reasoning skills to analyze the various relationships among the three unknown quantities. The latter of these two strategies would be considered more algebraic even though the formal procedures of algebra may not be visible. Of the pre-

Table 1. Preservice Teachers' Success With Various Tasks Associated With The Area Of A Trapezoid Problem

	Number of PSTs (n = 30)*	
Tasks	Yes	No
1. Given labeled drawings of trapezoids, explain the methods for finding area.	27	3
2. Write an expression to describe each method.	25	5
3. Show that the expressions are equivalent.	26	4
4. Given an expression, make a drawing and use it to explain the method for finding area.*	6	13
5. Explain why one expression is equivalent to the other three.*	13	6
6. Find the area of a given trapezoid using each of the four expressions.*	17	2

* Only 19 students attempted Tasks 4-6, perhaps because these were offered as bonus questions.

service teachers in this investigation, 8 set up and solved a system of three equations, 13 used informal reasoning skills, and 9 relied on guess and check. All but 2 preservice teachers successfully solved the problem.

Functions and Modeling

The functions and modeling approach focuses on various representations of dependence relationships among real-world quantities (Bednarz, Kieran, & Lee, 1996). In each investigation, the class modeled real-world situations and examined how the change in one variable related to change in a second variable. For each situation, they explored connections among tables, graphs, and equations. The responses to quiz questions taken from *Connected Mathematics* (Lappan et al., 1998a, b, & d) algebra modules provide evidence of some of preservice teachers' strengths and weaknesses regarding functions and modeling. Tables 2, 3, and 4 reveal preservice teachers' success with various tasks associated with linear, exponential, and quadratic functions, respectively.

Linear Functions.

Table 2 shows that more than a third of the preservice teachers had difficulty writing adequate explanations for how to determine a linear relationship from either a table or an equation. Additionally, more than a third failed to write an equation to match the graph of a line.

Exponential Functions

As shown in Table 3, only a third of the preservice teachers adequately described the exponential pattern of change in a given table. When given the equation $y = 2(3^x)$ only three preservice teachers correctly identified 200% as the rate of growth. Most preservice teachers said the rate of growth was 300%, evidently confusing the "rate of growth" with the "growth factor," even though the curriculum module carefully distinguishes between the two concepts.

Table 2. Preservice Teachers' Success With Various Tasks Associated With Linear Functions

	Number of PSTs (n = 30)	
Tasks	Yes	No
1. Recognize linear relationships from tables.	28	2
2. Explain how you can recognize a linear relationship from a table.	18	12
3. Recognize linear relationships from equations.	27	3
4. Explain how you can recognize a linear relationship from an equation.	17	13
5. Recognize linear relationships from graphs.	30	0
6. Given a graph, write an equation.	18	12
7. Explain how you can recognize a linear relationship from a graph.	28	2
8. Find an equation passing through two points on a graph.	26	4
9. Given a point and the slope, find an equation of the line.	26	4
10. Given a situation, make a table.	28	2
11. Given a situation, make a graph.	29	1
12. Write an equation to describe the situation.	26	4

Table 3. Preservice Teachers' Success With Various Tasks Associated With Exponential Functions

	Number of PSTs (n = 30)	
Tasks	Yes	No
1. Given a exponenetial situtation, create a table.	23	7
2. Describe the pattern of change in a table.	10	20
3. Given a exponential situation, write an equation.	26	4
4. Given an exponential equation, fill in a table.	30	0
5. Given two equations, determine in which equation the y value increases at a faster rate.	30	0
6. Given an exponential equation, determine the initial value of the function.	28	2
7. Given an exponential equation, determine the rate of growth.	3	27
8. Given tables, distinguish between a constant rate of increase and a percentage rate of increase.	26	4

Quadratic Functions

As Table 4 indicates, all but three preservice teachers were able to draw and label the rectangle represented by a quadratic equation, but a third of the preservice teachers failed to label the area of the rectangle. A more serious difficulty appeared in Tasks 9 and 10, in which preservice teachers struggled to extend a geometric pattern and to write the equation representing the pattern.

Preservice Teachers' Focus on Developing Children's Algebraic Reasoning

The data sources for assessing preservice teachers' focus on the development of children's algebraic reasoning were: (1) their responses to questions about "A Foundation for Early Algebraic Reasoning in the Early Grades" (Yackel, 1997) and (2) their analyses of the Chickens problem interview.

Responses to Yackel Article

The Yackel (1997) article seemed to provide preservice teachers with a language for describing algebraic reasoning. After reading the Yackel article, 16 of the 28 preservice teachers who completed the article response mentioned "patterns" or "relationships" in their answer to the following: "Describe the type of thinking that forms the foundation for algebraic reasoning in the elementary grades. How is this type of

Table 4. Preservice Teachers' Success With Various Tasks Associated With Quadratic Functions

	Number of PSTs (n = 30)	
Tasks	Yes	No
1. Recognize quadratic relationships from equations.	23	7
2. Explain how you can recognize a quadratic relationship from an equation.	28	2
3. Draw and label a rectangle represented by a quadratic equation.	27	3
4. Label the areas of the rectangle represented by a quadratic equation.	19	11
5. Recognize quadratic relationships from tables.	29	1
6. Explain how you can recognize a quadratic relationship from a table.	24	6
7. Write an equation for each table that represents a quadratic relationship.	24	6
8. Given a situation, determine a specific value of the quadratic function.	26	4
9. Extend a quadratic geometric pattern.	18	12
10. Write an equation to describe a quadratic geometric pattern.	15	15

thinking different from numerical reasoning?" Interestingly, though, the remaining 12 preservice teachers used language that suggested they equate algebraic reasoning with the process of problem solving (not to be confused with a problem-solving approach to algebra). In their descriptions of algebraic reasoning, these 12 preservice teachers mentioned such things as the importance of recognizing multiple solutions and having students explain their solution strategies, but did not refer to anything specifically algebraic.

Chickens Problem Interview

Yackel (1997) places emphasis not on the content of algebra, but rather on "the underlying thinking and reasoning of the students"(p. 276). Thus, in order for preservice teachers to develop a coherent view of algebraic reasoning, it is important to

provide opportunities for them to examine children's mathematical thinking. Indeed, Fennema and Franke (1992) suggest that knowledge of students' cognitions is more valuable to teachers than knowledge of learning theories. McGowen and Davis (2001) suggest that a way in which preservice teachers can become more effective teachers of early algebra is by seeing children solve problems with which they themselves have struggled. The Chickens problem interview was designed to serve this purpose.

The instructions asked preservice teachers to: (1) describe the student's work with the problem, noting questions asked and strategies attempted, and (2) discuss conclusions about the child's mathematical thinking. Unfortunately, because the instructions did not ask preservice teachers to focus on "algebraic reasoning," it is not appropriate to analyze their descriptions of algebraic reasoning. However, 13 of the 28 preservice teachers who completed this assignment carefully described the child's solution and discussed aspects of the child's algebraic reasoning. Another 9 preservice teachers mentioned algebraic reasoning, but did not provide a specific description. Finally, 6 preservice teachers focused entirely on other issues such as the child's attitude toward mathematics. The instructions also asked preservice teachers to reflect on the interview, describing what they would do differently another time. Eleven preservice teachers responded by saying they would ask more and better questions. Four said they would provide less help, and two said they would provide more help. Four preservice teachers said they would like to interview more children to get a broader perspective.

Preservice Teachers' Views about the Nature of Algebra

The data sources for analyzing preservice teachers' views about the nature of algebra were their initial and terminal responses to "What is algebra?" Preservice teachers' responses to the question, "What is algebra?" were assigned to the following categories: symbolic, problem solving, generalization, and functions. The latter three categories are derived from the three conceptual approaches to algebra adapted from Bednarz, Kieran, and Lee (1996). For coding purposes, *symbolic* responses merely listed various symbols and operations, *problem solving* focused on solving equations, *generalization* mentioned patterns or relationships, and *functions* included the word "functions" or mentioned connections among tables, graphs, and equations. Table 5 shows preservice teachers' responses on the first and last days of class. At the end of the course, none of the students indicated a symbolic view of algebra, instead expressing some combination of generalization, problem solving, and function definitions.

Conclusion

The results of this investigation show that exemplary curriculum materials developed for middle school students present mathematical challenges for some preservice elementary and middle school teachers. Preservice teachers in this investigation were challenged by situations involving generalization, problem solving, and functions. Although most found answers for problems involving linear relationships, the preser-

Table 5. Categories Of Preservice Teachers' Responses To The Question, "What Is Algebra?"

	Definition of Algebra*			
Data Source	Symbols	Generalization	Problem Solving	Functions
Number of PSTs (n = 30)				
Initial Responses	5	5	20	0
Terminal Responses	0	14	16	15

* Some preservice teachers' responses reflected more than one definition of alge

vice teachers were often unable to explain their thinking. They had difficulty answering questions about the rate of change of exponential and quadratic functions. However, they were sometimes bored by materials designed for elementary and middle school students. Preservice teachers often struggle to clearly communicate their understanding of the mathematical relationships embedded in the activities, but they do not always need to explore the same concept from as many angles as are presented in the well-developed materials. We are compelled to use the materials judiciously, selecting critical elements of the activities but avoiding over-repetition.

The interview situation with children proved to be an effective tool for helping preservice teachers begin to focus on children's mathematical thinking, and exemplary curriculum materials offer engaging problems for this purpose. Many of the preservice teachers in this investigation did not provide evidence that they understand the difference between algebraic reasoning and problem-solving skills. Even more discussion may be needed to help preservice teachers strengthen their understanding of what constitutes algebraic reasoning, and more than one of these interview experiences may be necessary to help preservice teachers shift their focus from children's problem-solving behaviors to children's algebraic reasoning.

As they experience exemplary curriculum materials from a learners' point of view, preservice teachers can begin to think about them from a teacher's perspective. It is important to take time to discuss the philosophy that guided the development of reform curricula so that preservice teachers will understand why these materials are so different from the textbooks with which they are more familiar. With guidance, they may come to appreciate the potential of the new curricula for developing children's mathematical thinking.

Before they can successfully promote algebraic thinking in their own classrooms, preservice teachers need to understand algebra as a way of thinking, a way of working with the patterns that occur every day. A comparison of their initial and terminal

responses to the question "What is algebra?" suggests that this group of preservice teachers broadened their views about the nature of algebra. Because their terminal definitions of algebra reflect the three conceptual approaches to algebra adapted from Bednarz, Kieran, and Lee (1996), it would seem that these approaches do indeed serve as a meaningful structure for framing the course. Providing an opportunity to explore exemplary elementary and middle school curricula in the context of a college algebra course has helped preservice teachers develop a dynamic vision of algebra they can take with them into the classroom.

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